The Mathematics of Gambling

with Related Applications

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Gambling

"**Gambling: the sure way of getting nothing for something**"
-Wilson Mizner

"No wife can endure a gambling husband unless he is a steady winner"
-Thomas Dewar


Motivation and Goals

- Accumulate information to give you an edge
- Use that information to the best of your ability
- Be successful in an uncertain environment

Outline

1. **Gaining an Edge**
2. **Avoiding Gambler's ruin**
3. **Bigger Picture**

Section 1

Gaining an Edge

Claude Shannon

Creator of Information theory

"I visualize a time when we will be to robots what dogs are to humans, and I'm rooting for the machines"

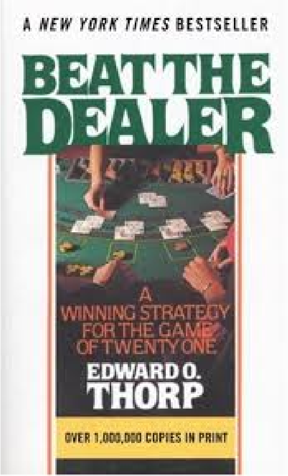
Claude Shannon- Life after Info Theory

Theseus the maze-solving mouse
(Machine Learning)

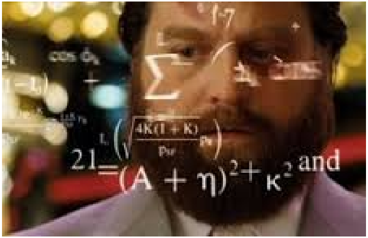
The "ultimate machine"

Edward O. Thorp

Father of the wearable computer

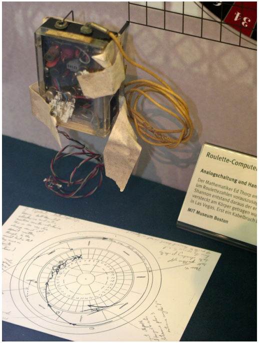


Why Don't Casinos change the Rules?



1. They tried
2. Most card counters are bad
3. Knockout dealers
4. You may be banned
5. Or worse

Approximate the quadrant the ball will land in. Bets placed for a second or two after wheel is spun.



Built in Shannon's basement and kept a secret Edward Thorp now known as the father of the wearable computer

Section 2

Gambler's Ruin

Coin Flip Game

Fair coin: 50-50 chance heads or tails

For some reason I am giving amazing odds of 3 dollars for every 1 dollar if the coin lands on heads

Imagine you have a dollar and want to play the game: **how much would you bet?**

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How about 10 dollars?

How to Invest

Bet all your Money

$$E\left[W^{n+1}\right] = \frac{3}{2} W^n$$

The problem is gambler's ruin: the coin eventually lands on tails

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Long-term investment growth

$X_n = X_0 g_1 g_2 \ldots g_n$

$X_n = X_0 2^{\log(g_1)+\log(g_2)+\ldots\log(g_n)}$

Central limit theorem says sum independent random ≈ mean:

$$R_{\max} = \max_{\mathbf{b}} E\left[\log g\right]$$

Optimal Betting

Maximize expected log of growth!

Say we bet a fraction of our money $b \in [0, 1]$ on heads

$$\log g = log(Ob + (1 - b)) \qquad \Pr = p$$
$$\log g = log(1 - b) \qquad \Pr = 1 - p$$

Odds I'm giving are 3 for 1 so $O = 3$, $p = 1/2$

$$max_b E[\log g] = \max_b [p \log(Ob + (1 - b)) + (1 - p) \log(1 - b)]$$

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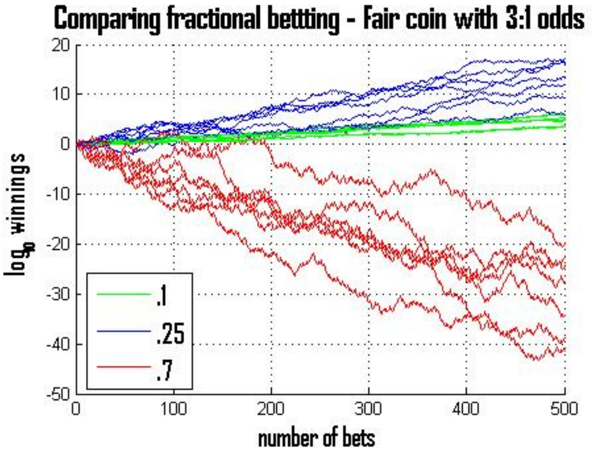
$$b^* = \frac{p - \frac{1}{O}}{1 - \frac{1}{O}}$$

Optimal Betting Simulation



In 500 games, you become a billionaire with the optimal strategy, a millionaire with the low risk strategy. Interestingly, even with the odds in your favor, you loose all your money if you bet too aggressively.

Optimal Betting for Blackjack

$O = 1/2$: Dealers give you 2 for 1 if you win (usually)

If the deck has a high count you have an edge: $p > 1/2$

$$b = 2p - 1$$

For a moderately high count $p = .51$ so bet 2 percent of your money

Section 3

Bigger Picture

Kelly Gambling

A beautiful theory relating information theory to gambling. Imagine a horse-race with n horses and odds o_i. If the true probability of each horse winning is p_i. Say you bet a fraction b_i of your money on each horse, then

$$R = E[\log(g)] = \sum_i p_i \log(b_i o_i)$$

$b_i^* = p_i$

$$D(p, q) = \sum_i p_i \log(p_i/q_i)$$

D measures the difference between two probability distributions

Money grows as $2^{mD(p,q)}$, if you bet in m races! $q_i = \frac{1}{o_i}$. Discrepancy between reality and dealer's beliefs can be exploited.

Intuition and Application

Diversify your bets: both hedging against disaster and maximizing growth rate. (Hedging)

Bet your beliefs

Example: Lottery, Securities

Information Theory: Compression

Intuition: The amount of predictability in a message tell you how much you can compress it. The higher the entropy of a language the less predictable

$$H = \sum_i -p_i \log p_i$$

Example

Imagine you want to make a language more efficient than English, but with 2 letter.

Take a dictionary and make a conversion chart

More probably words should have smaller lengths

Information Theory: Transmission

Intuition: Noise is corrupting your messages. Therefore you may have to repeat your messages or add error correcting bits. How much longer do the messages need to get?

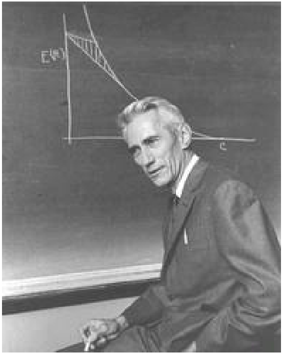
R=fraction of the message that is informative

$$R \leq I(X, Y) = D(p(x, y) || p(x)p(y))$$

X is input and Y is corrupted output. You are limited by how dependent the corrupted and original signal are on each other. If they are independent, you can't send messages!

Conclusions and Wrapping up

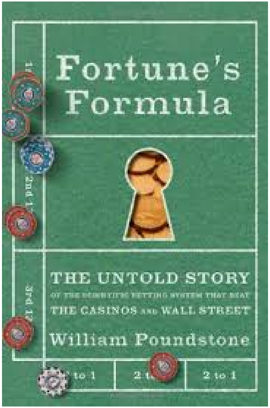
You need to find an edge to make money (bet probably not in Vegas)
Even with an edge, you have to bet appropriately to make money

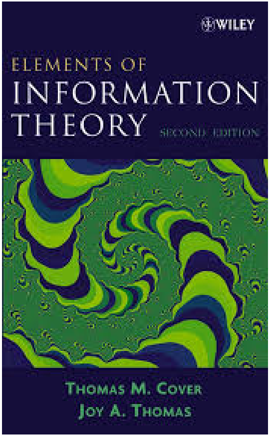


Made a 28 percent return on his portfolio: more than Warren Buffet. Continued Tinkering: Chess, Rubik's cube, etc.

Made an average of 20 percent return on investments over 25 years. Hedge Fund Manager - Currently President of Edward O. Thorp and Associates

References


Fortune's Formula
THE UNTOLD STORY
THE CASINOS AND WALL STREET
William Poundstone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