Introduction to Probability: Poker

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Splash!

Outline

- Motivation
- Coin Flips
- Dice
- Lottery
- Poker

Motivation

- Predicting things we don't know
  - Insurance
  - Finance
- Scientific Calculations
  - Simulations
- Computer Graphics
  - Pixar
- Poker

Notation

- E, Event
- P(E), Probability of an event happening
- $0 \leq P(E) \leq 1$
- Union, $E \cup F$
- Intersection, $E \cap F$
- Factorial, n! = n*(n-1)*(n-2)*…*3*2*1

Coin Flips

- Thomas Jefferson or Buffalo
- P(Thomas Jefferson) = P(J) = 0.5
- P(Buffalo) = P(B) = 0.5

Images: http://www.usmint.gov/about_the_mint/coinLibrary/

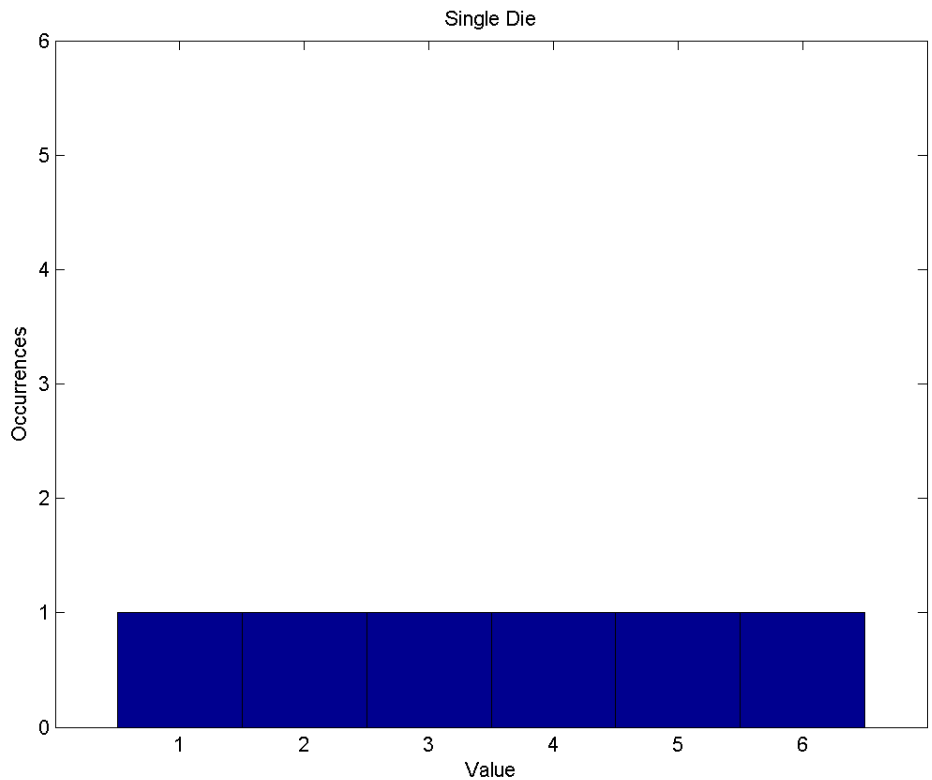
Multiple Coin Flips

- P(JJ) = P(J)*P(J) = 0.25
- P(JB) = P(J)*P(B) = 0.25
- P(BJ) = P(B)*P(J) = 0.25
- P(BB) = P(B)*P(B) = 0.25

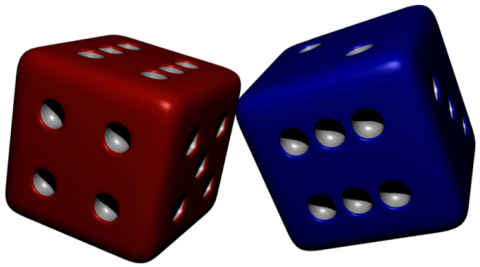
- P(E then F) = P(E)*P(F)

Die

- 6 sides, all equal probability



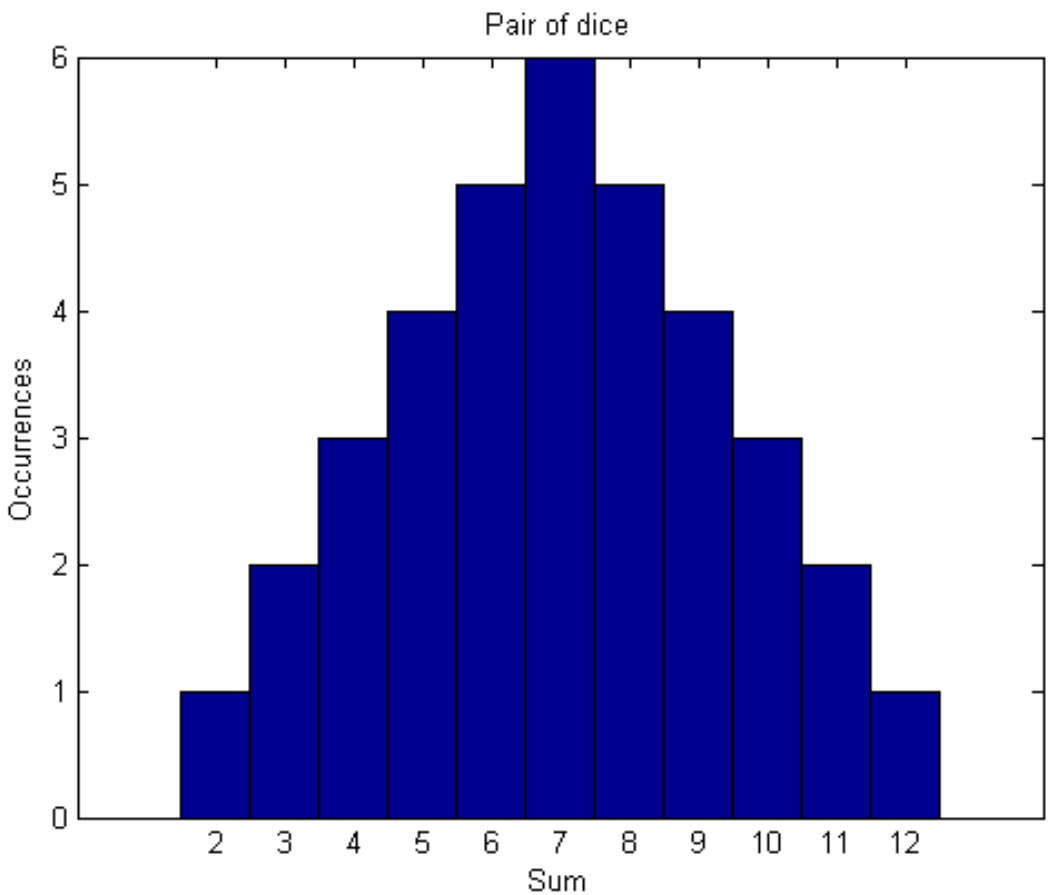
Two Dice



- Sum

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

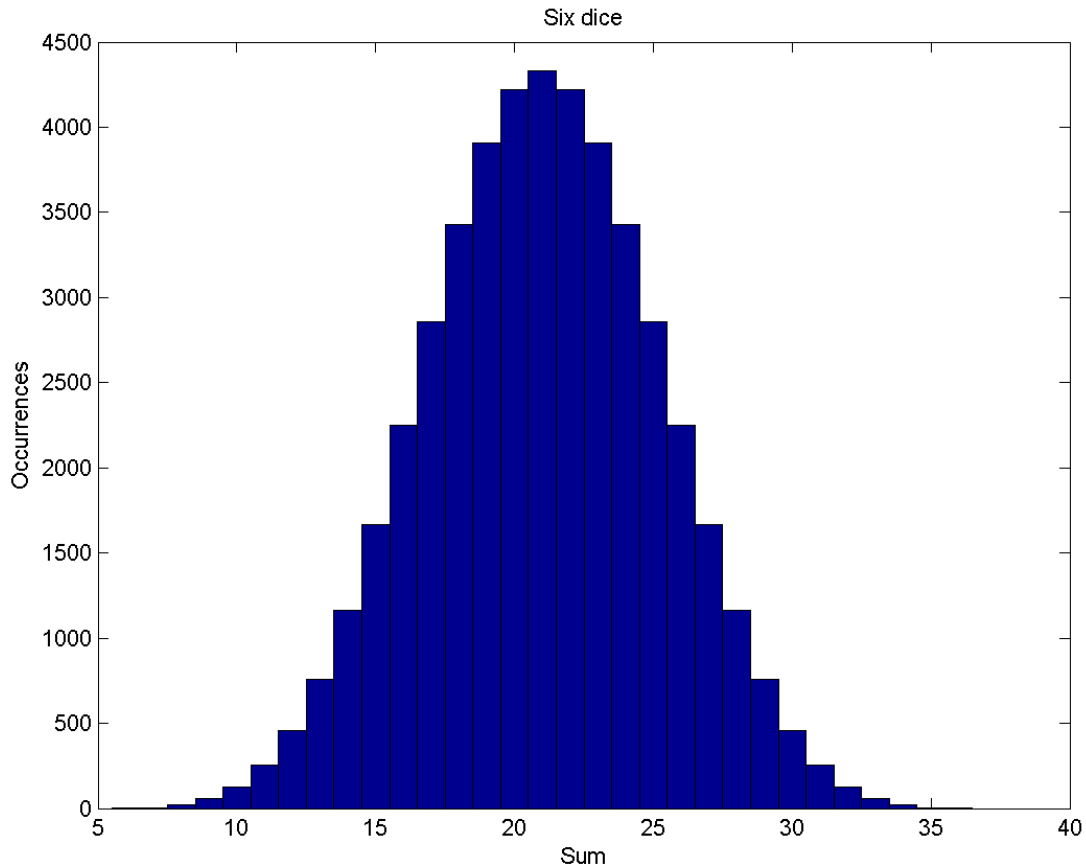
Two Dice



Question

- What do I expect to get if I roll 6 dice?

6 dice


Six dice
Occurrences
Sum
4500
4000
3500
3000
2500
2000
1500
1000
500
0
5
10
15
20
25
30
35
40

Mega Millions Lottery

- 5 Numbers from 1 to 56
- 1 Number from 1 to 46

P and C Notation

- 10p3 = 10*9*8 = 10!/7!
- $$10c3 = \frac{10!}{3!7!} = \binom{10}{3} = \binom{10}{7} = \frac{10*9*8}{3*2*1}$$

Mega Millions Lottery

- 5 Numbers from 1 to 56
- 1 Number from 1 to 46
- Jackpot
  - 1 out of (56c5*46)
  - 175,711,536
- Expected Value

$$\sum P(E) * (\text{winnings from event})$$

http://www.megamillions.com/howto/

When should I play the lotto?

- $1 = P(Jackpot)*Jackpot + Everything else
- $1 = P(Jackpot)*Jackpot + $0.18
- $0.8181 = P(J)*Payout
- Payout = $0.8181/P(J)
- Payout = $143,752,177

MATCH ●		MATCH ●	PRIZE*	CHANCES
5	+	1	Jackpot	1 in 175,711,536
5	+	0	$250,000	1 in 3,904,701
4	+	1	$10,000	1 in 689,065
4	+	0	$150	1 in 15,313
3	+	1	$150	1 in 13,781
3	+	0	$7	1 in 306
2	+	1	$10	1 in 844
1	+	1	$3	1 in 141
0	+	1	$2	1 in 75
Overall chances of winning a prize:				1 in 40

Image: http://megamillions.com/howto/

Deck of Cards

- 52 Cards
- 4 Suits
  - Diamonds, Clubs, Hearts, Spades
- 13 Ranks
  - Ace, 2 through 10, Jack, Queen, King

Poker

- 5 card hands
- Texas Hold'em
- Many Variations

Hand

- Any 5 card combination
- Number of ways cards can be passed out
  - $52 \times 51 \times 50 \times 49 \times 48 = 52p5 = 311{,}875{,}875$
- Number of unique hands
  - $52!/47!/5! = 52c5 = 2{,}598{,}960$

Poker Hands

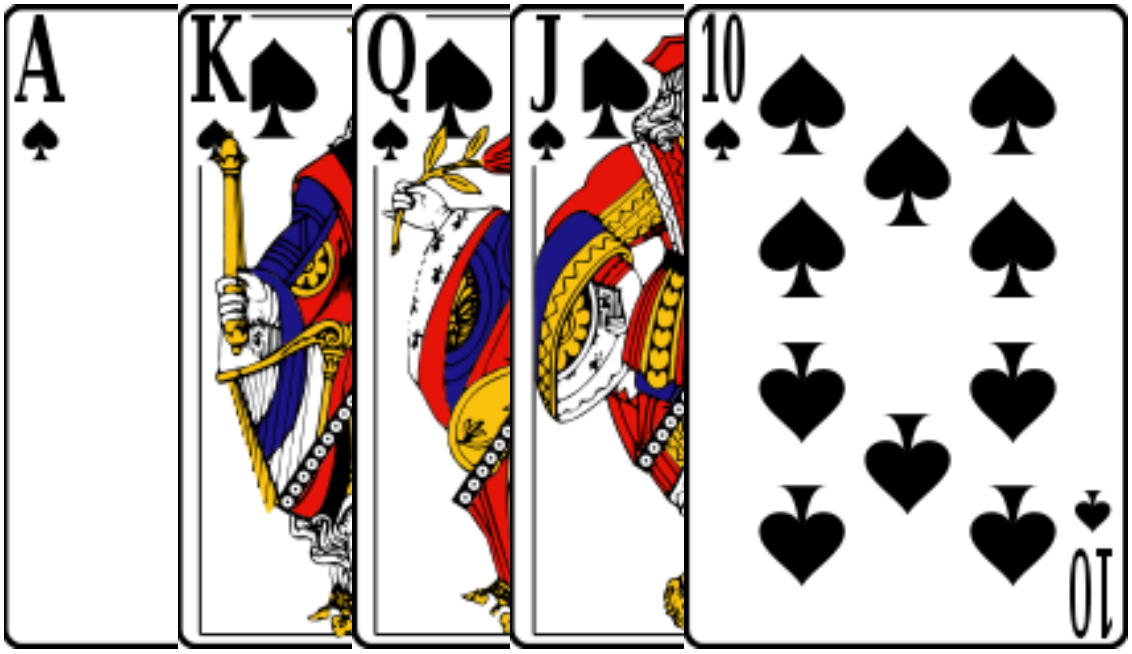
- Junk
- Pair
- Two Pair
- Three of a Kind
- Straight
- Flush
- Full House
- Four of a Kind
- Straight Flush
- Royal Flush

Let's Calculate Some Probabilities

- Start with rarest
  - Because a royal flush and straight flush are both flushes

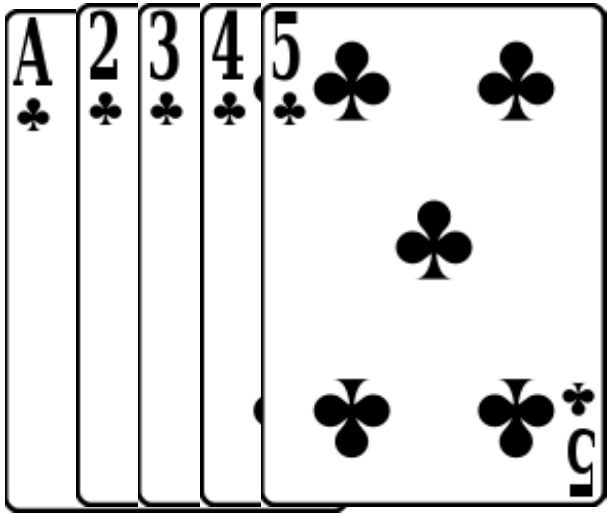
Royal Flush

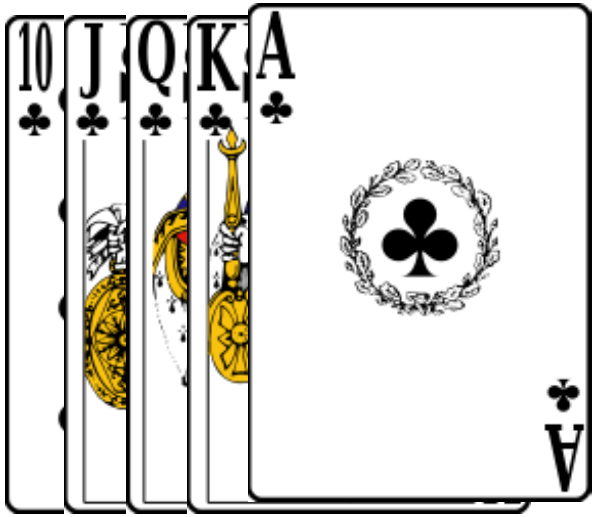
- A, K, Q, J, 10 all in a single suit
- 4 Hands



Straight Flush

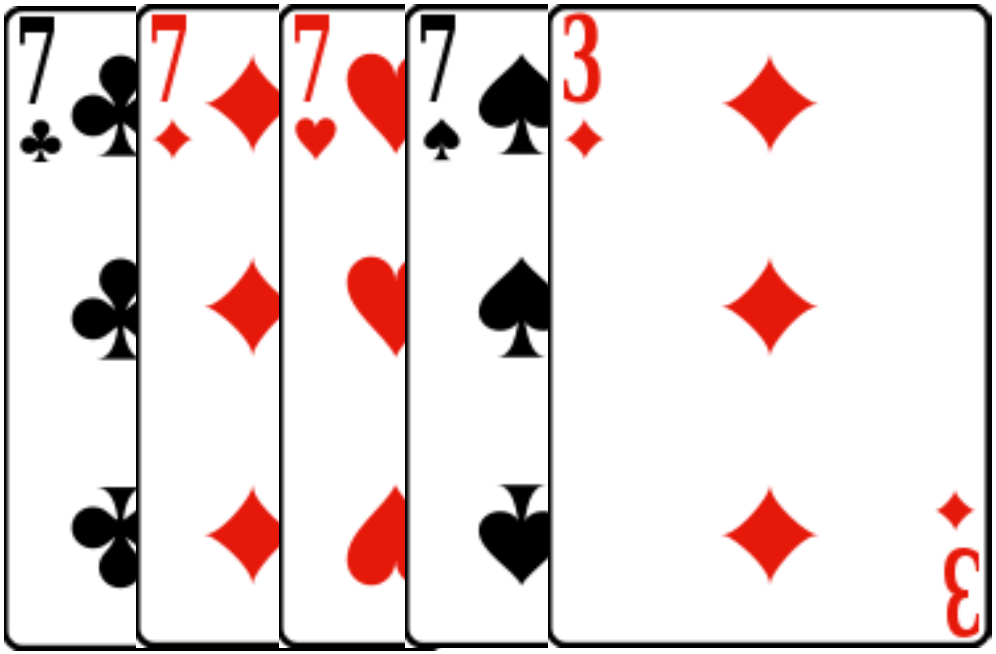
- 10 by rank, 4 suits
- 4 are royal flushes
- 36 Total





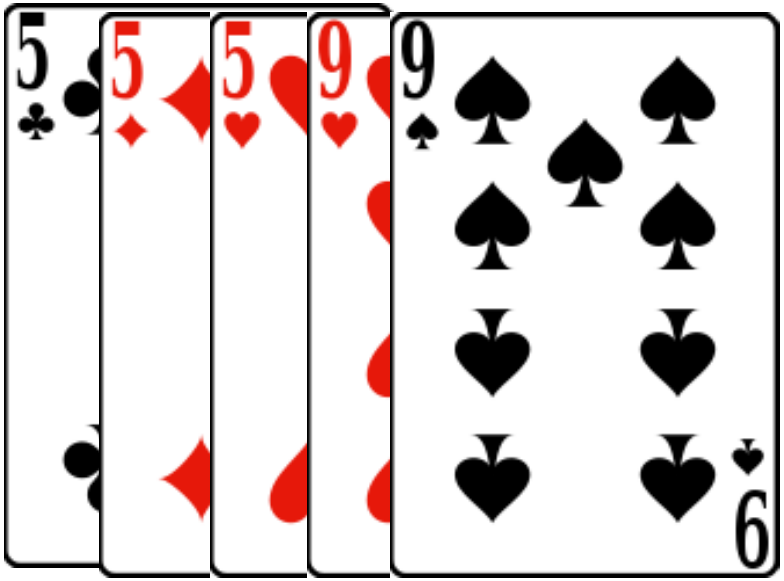
Four of a Kind

- 13 ranks, 48 extra cards
- 624 Total



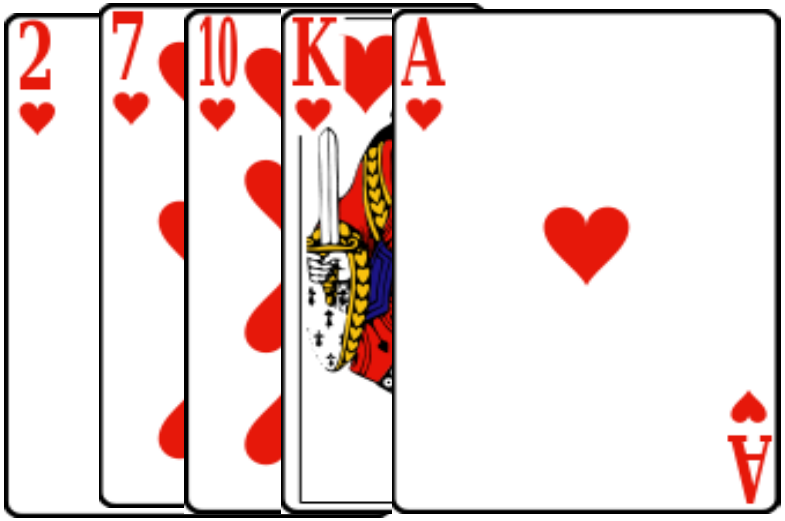
Full House

- Pick 3 from 13 ranks, and 2 from 12 remaining ranks
- 13*4C3*12*3C2 = 3,744 Total



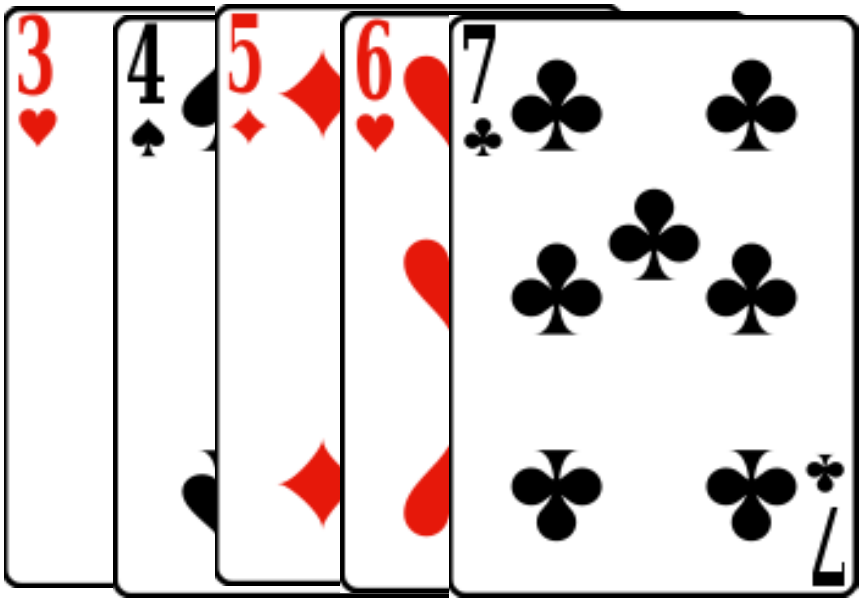
Flush

- 4 Suits, choose 5 from 13 ranks, subtract straight flush
- 4*13C5 - 40 = 5,108



Straight

- 10 starting ranks, each card can be a different suit minus straight flush
- 10*4^5 – 40 = 10,200

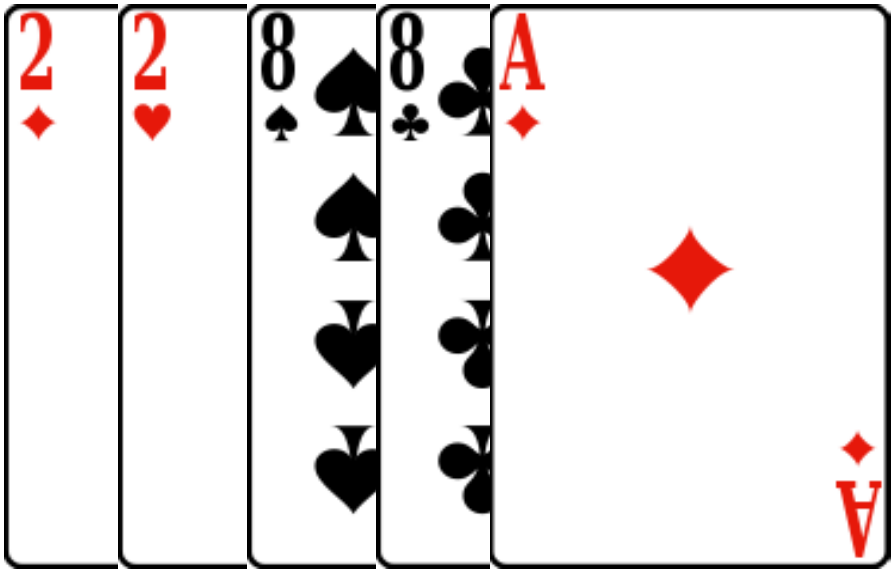


Three of a Kind

- 13 ranks, choose 3 of 4 suits, choose 2 of the 12 remaining, each can be different suit
- 13*4C3*12C2*4^2 = 54,912

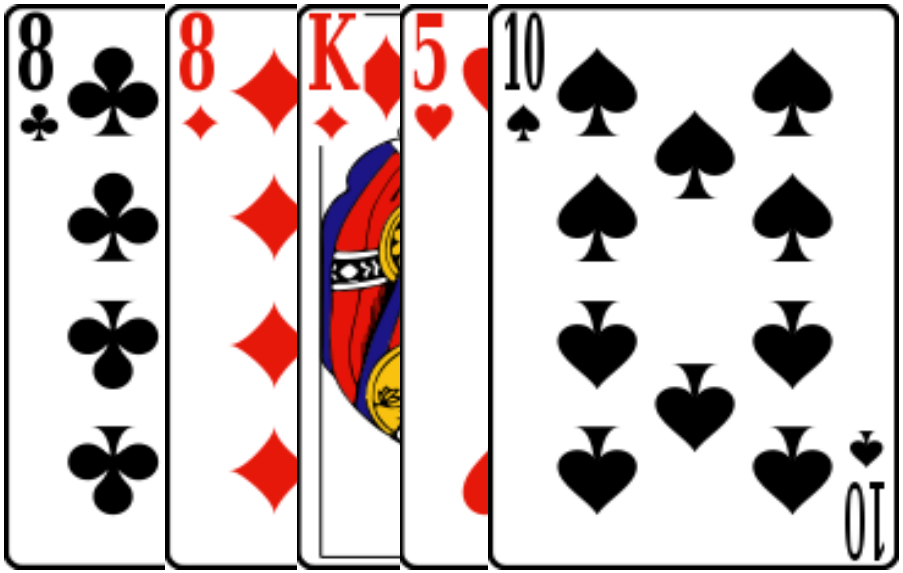
Two Pair

- Choose 2 of 13 ranks, Choose 2 of 4 in rank, Choose from 11 remaining ranks, any suit
- 13C2*(4C2)^2*11*4 = 123,552



Pair

- 13 ranks, choose 2 of 4 suits, Choose 3 of remaining 12 ranks, each can be different suit
- 13*4C2*12C3*4^3 = 1,098,240



Junk

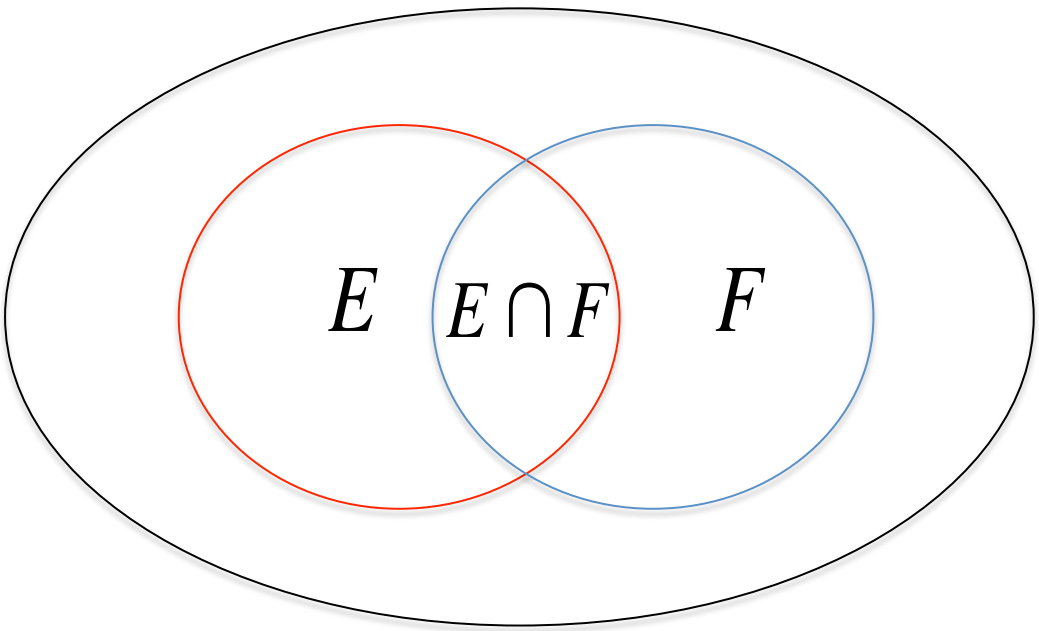
- Total Hands minus everything else
- 1,302,530 Total

Summary of Poker Probabilities

Hand	Occurrences	Probability
Royal Flush	4	0.00000154
Straight Flush	36	0.00001385
Four of a Kind	624	0.00002401
Full House	3,744	0.00144057
Flush	5,108	0.00196540
Straight	10,200	0.00392465
Three of a Kind	54,912	0.02112845
Two Pair	123,552	0.04753902
One Pair	1,098,240	0.42256903
Junk	1,302,540	0.50117739
Total	2,598,960	1

Conditional Probability

- Calculate new probabilities given information
- P(E | F), probability of E given F happened



$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

Bayes' Theorem

- Used in Bayesian email Spam filters

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Summary

- Coins
  - Multiple Events
- Lottery
  - Expected Value
- Poker
  - Combinatorics
- Conditional Probability

References

- http://david.bellot.free.fr/svg-cards/
- http://en.wikipedia.org/wiki/Poker_probability
- http://en.wikipedia.org/wiki/Poker_probability_(Texas_hold_'em)
- A first course in probability by Sheldon M. Ross

Face Up Poker

- Demo

http://revision3.com/scamschool/pokerface